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SOME STUDIES IN RANDOM FIXED POINT THEOREMS FOR METRIC SPACES

By

Mr. Bhosale Pavan Dnyanoba

(Research Student)

Dayanand Science College, Latur

Latur-413512 (Maharashtra State), India

Email: pavanrajed80@gmail.com Ph:9718470749

Under the Guidance of

Dr. S. S. Bellale

Dept. of Mathematics

Dayanand Science College, Latur-415312

Maharashtra State (INDIA)

Email:- sidhesh.bellale@gmail.com

Phone:- 94054 17417

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Research Centre

Dept. of Mathematics

Dayanand Science College, Latur

Dist.: Latur-413531 (MS), India

Phone No.: 02382-221149

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Abstract of proposed work plan

In the present research work we shall study some new theorems and concepts related to Random fixed points in metric spaces by using random operator. The theorems obtained here can be used as concepts in metric space. Also we shall study of compete metric space, Banach fixed point theorem and use these concepts in present research work.

1. Introduction:

In 1922, the polish mathematician Stefan Banach established remarkable fixed point theorem, the famous contraction principle, which is one of the most important result of analysis. It is most widely applied fixed point result in the areas of the mathematics and applications. It required the structure of complete metric space with contractive condition on the map which easy to test in many situations. It has been generalized in many different directions. Moreover, the proof of the Banach contraction principle gives a sequence of approximate solutions and useful information as regards the rate of convergence toward the fixed point. This is very important since a fundamental principle both in mathematics and computer science is iteration. Particularly, fixed point iterations and monotone iterative techniques are the core methods when solving a large class of abstract and applied mathematical problems and play an important role in many algorithms.

As stated by Brown R. F. "In mathematics, a fixed –point theorem is a result saying that a function F will have at least one fixed point (a point x for which F(x) =x) under some conditions on F that can be stated in general terms." Moreover, a contraction mapping has the one fixed point. Under this, the Banach fixed-point theorem states that each contraction mapping on a non-empty complete topological space which contains a distinct fixed points.

Random operator theory is needed to study various class of random equations. Indeed in many cases, the mathematical model or equations used to describe the phenomena in biology, physics, engineering, and systems sciences contain certain parameters or coefficients which have specific interpretations, but whose values are unknown. Therefore, it is more realistic to consider such equations as random operator equations.

The goal of this paper is to establish a random version of some fixed point theorems for metric spaces. In this paper we extend the fixed point results, theorems using the concept of metric spaces and study of extended theorems on the fixed point theory. Also I will try to find out fixed point theorem with applications which will be new generalization on our research work.

2. Review of Literature and Development on the subject (previous work done in the relevant area);

In wide range of mathematical, computational, Economical, modeling and engineering problems the existence of a solution to a theoretical or real world problem is equivalent to the existence of a fixed point for a suitable map or operator. Fixed points are therefore of paramount importance in many areas of mathematics, Sciences and engineering

In Polish spaces, i.e., separable complete metric spaces, random fixed point theorems for contraction mappings were proved by Speak and Hang etc. Mukherjea [1] (P-106) gave a random fixed point theorem of Schauder type on an atomic probability measure space. Then Prakasa Rao extended this result and obtained a theorem of Krasnosel'skii type on a same measure space. Recently, Bharucha-Reid generalized results of Mukherjea and Prakasa Rao to the cases on general probability measure spaces. We refer to Bharucha-Reid for a survey of related results. In we obtained a random fixed point theorem for a multivalued con-traction mapping in a Polish space. In this paper, we give several random fixed point theorems for various single valued mappings, e.g.,

condensing or non-expansive mappings, etc., in Banach or Hilbert spaces on general measurable spaces.

The Random fixed point theory is an important and major topic of the Metric spaces and functional analysis and finds some nice applications to other branches of Mathematics as well as find some new theorems for fixed point theory.

The theory itself is a beautiful mixture of analysis (pure &applied) topology and geometry. Over the last 60 years or so the theory of fixed points has been revealed as a very powerful and important tool in the study of metric spaces.

Many situation in the study of non-linear equations, calculus of variations, partial differential equation, optimal control, and inverse problem formulated in terms of fixed point problems.

3. Objective of Research / Proposed Hypothesis:

The present work highlight of the following valuable achievements:

- To discuss the fixed points in Metric Space.
- To discuss the Random fixed point theorems in Metric Spaces.
- To discuss the fixed points in contraction mapping.
- To discuss the fixed points for Banach and F-Suzuki contraction mapping in complete Metric Space and complete b-metric space.
- To discuss the fixed points for contraction in complete metric space
- To study of Banach contraction mapping.
- To study fixed points for bounded metric space.
- To study the generalization of fixed point theorems for Banach and F-Suzuki contraction mapping

- To study the generalization of fixed point theorems in Banach Space and Metric Space.
- To study the application of fixed point theorems for Banach and F-Suzuki contraction mapping in complete metric space and complete b-metric space.

4. Importance of Study / Society Application:

- **International Status:**

The importance of random fixed point theory lies in its vast applicability in probabilistic functional analysis and various probabilistic models. The introduction of randomness however leads to several new questions of measurability of solutions, probabilistic and statistical aspects of random solutions. It is well known that random fixed point theorems are stochastic generalization of classical fixed point theorems what we call as deterministic results. Random fixed point theorems for random contraction mappings on separable complete. Metric spaces were first proved by Spaˇcek L and Hanˇs . The survey article by Bharucha-Reidˇ [1] in 1976 attracted the attention of several mathematicians and gave wings to this theory. Itoh extended Spaˇcek's and Hanˇs's theorems to multivalued contraction mappings. Random fixed point theorems with an ˇ application to Random differential equations in Banach spaces are obtained by Itoh . Sehgal and Waters had obtained several random fixed point theorems including random analogue of the classical results due to Rothe . In recent past, several fixed point theorems including Kannan type Chatterjeea and Zamfirescu type have been generalized in stochastic version.

On the otherhand, Padgett used the random analogue of Banach fixed point theorem to analyze the Existence and uniqueness of random solution of a nonlinear stochastic integral equation of the Hammerstein Type of the form

$$x(t;w) = h(t;w) + \int_s k(t,s;w) f(s, x(s;w)) d\mu(s)$$

And proved several theorems. Achari and Saha continued to work on this application for more generalized random nonlinear contraction operators. Many mathematicians generalized the fixed point theory Banach S., Khan M. S., Khamsi M. A .and Swaleh M. Cric L. B., Jungck G. and Rhoades B. E, Branciari A, Browder F. E . Have worked in fixed point theory. In currently decades,Zead Mustafaand Brailey Sims developed nice area i.e. Metric space and Generalized many fixed points additionally.

- **National Status:**

Indian Mathematicians additionally operating within the fixed point theory in Metric Space, Complete Metric Space, Dislocated Metric Space etc... In those studies, extend the fixed point results of Wardowski, Wardowski and Dung, and Piri and Kuman by introducing a generalized F-Suzuki-contraction in metric spaces. In many branches of science, economics, computer science, engineering and the development of nonlinear analysis.

5. Methodology to be adopted:

- In the first stage of my research work I will try to build up the necessary background to understand the area of my research work and for this purpose, I will read the useful research textbooks.
- Survey of the research papers to collect the literature related to my research problem.
- Generalization of new theorems which will be new contribution in research.
- Fourth stage will be the new results and conclusion.

6. Year wise Plan of Work and Target to be Achieved:

- **First Year:** From different research centers, libraries and internet, we will study the basic literature related to the research problem. Researcher will attend and present some papers related to problem in some Journals and conferences. Also by rule of SRTMU, Nanded complete the course work in the present year. Also few libraries and institution like TIFR, IIS, ISI, IITs, and some universities will be visited for this purpose. To progress reports will be submitted at the end of this year.
- **Second Year:** To attend the conference, workshop and seminars in relevant topic and collect research papers for study for further research work. The elected material will be proceeded and adopt will be made to discover the finding in the direction on research methodology, tools, technics and guidance will be apply for new research problem.
- **Third Year:** In this stage we will study some fixed point theorems in Metric spaces and its generalizations for F-Suzuki contraction mapping and try to generalize them. Also to publish research papers in reputed journals and to find the Application of fixed point theorems in Complete Metric Space and the plan to submit the thesis to University.

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Signature

Research Scholar Supervisor **Signature of Research Student**